The coefficients A_n, B_n, C_n and D_n may now be evaluated by satisfying the boundary conditions at the air-glass and glass-plasma boundaries. These are

$$\text{At} \quad r = b \qquad \frac{\partial V_0}{\partial r} = \kappa_g \frac{\partial V_g}{\partial r} \quad \text{and} \quad V_0 = V_g$$

$$\text{At} \quad r = a \qquad \kappa_g \frac{\partial V_g}{\partial r} = \kappa_p \frac{\partial V_p}{\partial r} \quad \text{and} \quad V_p = V_g$$

where κ_g is the relative dielectric constant of the glass tube and κ_p the relative dielectric constant of the plasma. We are interested only in the potential in the plasma. Carrying out the process indicated above we find that

$$V_p = \left\{ \frac{-4E\kappa_g/a^2}{[(1+\kappa_g)(\kappa_p+\kappa_g)/a^2] - [(1-\kappa_g)(\kappa_p-\kappa_g)/b^2]} \right\} \cdot r \cos\theta.$$

The resonance condition occurs when the denominator is zero. From this and the fact that

$$\kappa_p = (1 - \omega_p{}^2/\omega^2)$$

we obtain

$$\left(\frac{\omega_p}{\omega}\right)^2 = 1 + \frac{\kappa_g\left[\frac{b^2}{a^2}(1+\kappa_g) - (\kappa_g - 1)\right]}{\frac{b^2}{a^2}(1+\kappa_g) + (\kappa_g - 1)}.$$

Thus

$$\kappa_{\text{eff}} = \kappa_g \frac{\frac{b^2}{a^2}(1+\kappa_g) - (\kappa_g - 1)}{\frac{b^2}{a^2}(\kappa_g + 1) + (\kappa_g - 1)}.$$

For the tube used, $a/b = 0.853$ and $\kappa_g = 3.78$ giving $\kappa_{\text{eff}} = 1.53$.

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Correspondence

Couplings in Direct-Coupled Waveguide Band-Pass Filters*

Couplings in direct-coupled waveguide band-pass filters can be described in two different ways. Cohn[1] describes couplings in terms of normalized susceptances of the coupling circuit elements (*e.g.*, posts or irises). Dishal[2] describes interstage couplings in terms of a coefficient of coupling between adjacent resonators. Although Cohn has briefly mentioned coefficients of coupling in his paper,[1] the interchangeability of the two methods of describing the couplings has not been clearly established.

The low-pass prototype of the direct-coupled waveguide band-pass filter is shown in Fig. 1.

$$K_{12} = \frac{\Delta f_{12}}{f_0} = \frac{1}{\omega_1'\sqrt{C_1 L_2}}\left(\frac{f_2 - f_1}{f_0}\right) \qquad (1)$$

* Received April 2, 1962, revised manuscript received, May 2, 1962.

[1] M. Dishal, "Dissipative band-pass filters," Proc. IRE, vol. 37, pp. 1050–1069; September, 1949.

[2] S. B. Cohn, "Direct-coupled resonator band-pass filters," Proc. IRE, vol. 45, pp. 187–195; February, 1957.

where

K_{12} = coefficient of coupling between resonators 1 and 2
f_0 = center frequency of filter
ω_1' = pass band edge of low-pass prototype
f_2 and f_1 = corresponding pass band edges of waveguide filter
C_1 and L_2 = circuit elements in low-pass prototype.

Δf_{12} is a coupling bandwidth that can be measured by simple experimental techniques. The utilization and measurement of this coupling bandwidth is described in detail by Dishal.[3]

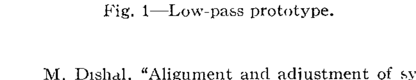

Fig. 1—Low-pass prototype.

[3] M. Dishal, "Alignment and adjustment of synchronously tuned multiple-resonant-circuit filters," Proc. IRE, vol. 39, pp. 1448–1455; November, 1951.

The lumped-circuit band-pass equivalent of the direct-coupled waveguide band-pass filter is shown in Fig. 2. This equivalency is usually satisfactory for narrow-bandwidth filters (*i.e.*, $\leqq 5$ per cent bandwidth).

$$B_{12} = \frac{1 - \frac{L^2}{C_1 L_2}}{\frac{L}{\sqrt{C_1 L_2}}} \cong \frac{1}{\frac{L}{\sqrt{C_1 L_2}}}$$

for narrow-bandwidth filters (2)

where

$$B_{12} = \frac{Z_0}{2\pi f_0 L_{12}} = \text{normalized susceptance of coupling circuit element.}$$

$$L = \frac{\pi}{2\omega_1'}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2\left(\frac{f_2 - f_1}{f_0}\right)$$

$$= \text{frequency variable} \qquad (3)$$

λ_{g0} = guide wavelength at filter center frequency
λ_0 = free space wavelength at filter center frequency.

Substituting (3) for L in (2)

$$B_{12} = \frac{1}{\frac{\pi}{2\omega_1'}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2\left(\frac{f_2 - f_1}{f_0}\right)\frac{1}{\sqrt{C_1 L_2}}}. \quad (4)$$

Combining (1) and (4):

$$B_{12} = \frac{1}{\frac{\pi}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 K_{12}}. \quad (5)$$

In the general case:

$$B_{ij} = \frac{1}{\frac{\pi}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 K_{ij}} \quad (6)$$

where

B_{ij} = normalized susceptance of coupling circuit element between ith and jth resonator

K_{ij} = coefficient of coupling between ith and jth resonator.

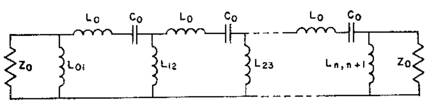

Fig. 2—Lumped circuit band-pass equivalent.

Eq. (6) can be employed to interchange normalized susceptances and coefficients of couplings for narrow-band direct-coupled waveguide band-pass filters. With appropriate modifications, this interchangeability can be extended to narrow-band lumped-circuit and coaxial band-pass filters.

It should also be noted that this paper is limited to interstage couplings. Input/output couplings have not been considered and will require a somewhat different analytical development.

Eq. (6) was independently derived and used by Sleven.[4]

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[4] R. L. Sleven, "Design of a tunable multi-cavity waveguide band-pass filter," 1959 IRE NATIONAL CONVENTION RECORD, pt. 3, pp. 91–112.

Contraphaseshifter*

SUMMARY

A novel combination of ring hybrid and variable power divider is shown to result in a three-terminal device having a single matched input and two outputs whose relative phase can be continuously varied independent of their amplitude.

* Received April 5, 1962.

INTRODUCTION

A contraphaseshifter is defined here to mean a device which first splits a signal into two parts and then provides a means by which the relative phase between the two output terminals can be varied. Such a device has obvious application to linear-phased arrays for which the relative phase between corresponding elements on opposite sides of the center of the array must be varied in order to scan the beam. A contraphaseshifter is often comprised of a number of simpler devices. For example, a fixed two-way power divider feeding a pair of mechanically ganged variable phase shifters driven in opposite directions may be regarded as a contraphaseshifter. It is the purpose of this paper to describe a novel combination of two well-known microwave components which together constitute a contraphaseshifter even though neither component is a variable phase shifter per se.

THEORY OF OPERATION

Consider the ring hybrid shown in Fig. 1(a). An incident voltage of magnitude V at terminal 1 produces outputs at terminals 2 and 3 given by $V_2 = V_3 = 0.707V$. Since terminals 1 and 4 are isolated from each other, we may also apply simultaneously to terminal 4 a second voltage of magnitude V' which we will take to be in quadrature with V. For this condition the voltages at 2 and 3 become

$$V_2' = j0.707V' \quad \text{and} \quad V_3' = -j0.707V'.$$

The total voltages at 2 and 3 are

$$V_{2T} = V_2 + V_2' = 0.707\sqrt{V^2 + V'^2}\, e^{j\tan^{-1} V'/V}$$

$$V_{3T} = V_3 + V_3' = 0.707\sqrt{V^2 + V'^2}\, e^{-j\tan^{-1} V'/V}.$$

Thus, the magnitudes of the voltages at 2 and 3 are equal to each other and dependent only on the sum of the squares of the voltages applied to 1 and 4, that is, dependent only on the total power into the ring hybrid. The relative phase of the voltages at 2 and 3, however, is dependent upon the ratio of V'/V. Thus:

$$\frac{V_{2T}}{V_{3T}} = e^{j2\tan^{-1} V'/V}. \quad (1)$$

To make a contraphaseshifter then, a second device is required which will

1) Split a signal into two quadrature components
2) Vary the magnitude of these components without varying their phase

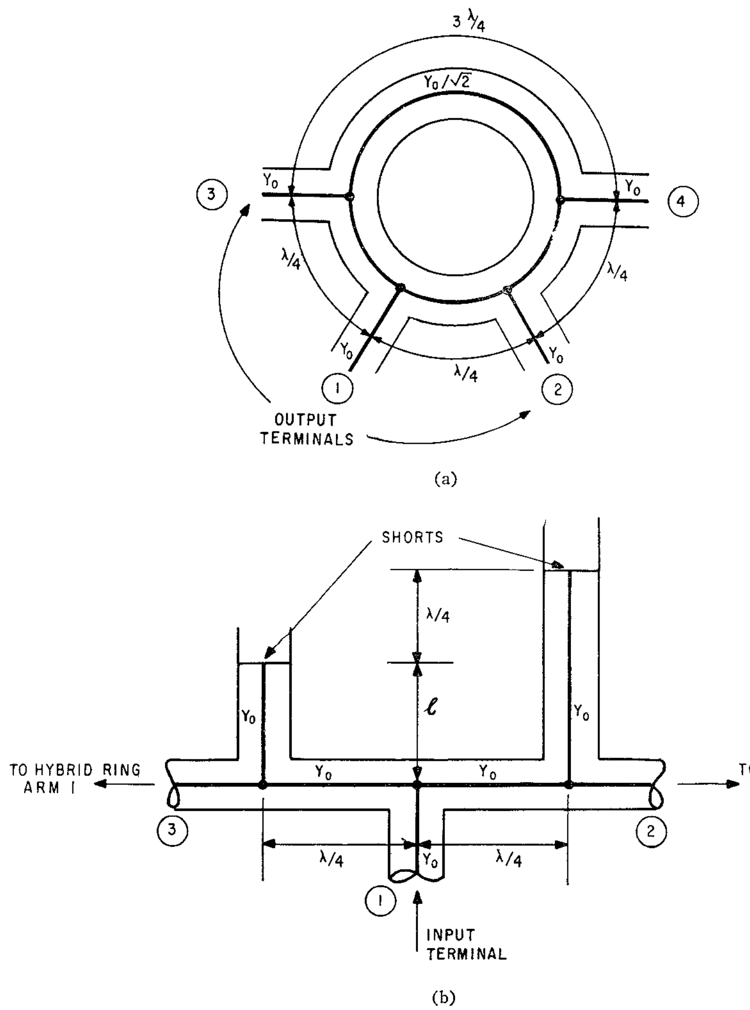

Fig. 1—Contraphaseshifter. (a) Hybrid ring. (b) Variable power divider.